Chapter Objective:

This chapter discusses exchange-traded currency futures contracts, options contracts, and options on currency futures.
Chapter Outline

• Futures Contracts: Preliminaries
• Currency Futures Markets
• Basic Currency Futures Relationships
• Eurodollar Interest Rate Futures Contracts
• Options Contracts: Preliminaries
• Currency Options Markets
• Currency Futures Options
Chapter Outline (continued)

- Basic Option Pricing Relationships at Expiry
- American Option Pricing Relationships
- European Option Pricing Relationships
- Binomial Option Pricing Model
- European Option Pricing Model
- Empirical Tests of Currency Option Models
Futures Contracts: Preliminaries

- A futures contract is *like* a forward contract:
  - It specifies that a certain currency will be exchanged for another at a specified time in the future at prices specified today.

- A futures contract is *different from* a forward contract:
  - Futures are standardized contracts trading on organized exchanges with daily resettlement through a clearinghouse.
Futures Contracts: Preliminaries

- Standardizing Features:
  - Contract Size
  - Delivery Month
  - Daily resettlement

- Initial Margin (about 4% of contract value, cash or T-bills held in a street name at your brokers).
Daily Resettlement: An Example

Suppose you want to speculate on a rise in the $/¥ exchange rate (specifically you think that the dollar will appreciate).

<table>
<thead>
<tr>
<th></th>
<th>U.S. $ equivalent</th>
<th>Currency per</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wed</td>
<td>Tue</td>
</tr>
<tr>
<td>Japan (yen)</td>
<td>0.007142857</td>
<td>0.007194245</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-month forward</td>
<td>0.006993007</td>
<td>0.007042254</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-months forward</td>
<td>0.006666667</td>
<td>0.006711409</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-months forward</td>
<td>0.00625</td>
<td>0.006289308</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Currently $1 = ¥140. The 3-month forward price is $1=¥150.
Daily Resettlement: An Example

- Currently $1 = ¥140 and it appears that the dollar is strengthening.
- If you enter into a 3-month futures contract to sell ¥ at the rate of $1 = ¥150 you will make money if the yen depreciates. The contract size is ¥12,500,000
- Your initial margin is 4% of the contract value:
  \[ \$3,333.33 = 0.04 \times ¥12,500,000 \times \frac{\$1}{¥150} \]
Daily Resettlement: An Example

If tomorrow, the futures rate closes at $1 = ¥149, then your position’s value drops.

Your original agreement was to sell ¥12,500,000 and receive $83,333.33

But now ¥12,500,000 is worth $83,892.62

\[ $83,892.62 = ¥12,500,000 \times \frac{\$1}{¥149} \]

You have lost $559.28 overnight.
Daily Resettlement: An Example

- The $559.28 comes out of your $3,333.33 margin account, leaving $2,774.05
- This is short of the $3,355.70 required for a new position.

\[
3,355.70 = 0.04 \times ¥12,500,000 \times \frac{1}{¥149}
\]

- Your broker will let you slide until you run through your maintenance margin. Then you must post additional funds or your position will be closed out. This is usually done with a reversing trade.
Currency Futures Markets

- The Chicago Mercantile Exchange (CME) is by far the largest.
- Others include:
  - The Philadelphia Board of Trade (PBOT)
  - The MidAmerica commodities Exchange
  - The Tokyo International Financial Futures Exchange
  - The London International Financial Futures Exchange
The Chicago Mercantile Exchange

- Expiry cycle: March, June, September, December.
- Delivery date 3rd Wednesday of delivery month.
- Last trading day is the second business day preceding the delivery day.
- CME hours 7:20 a.m. to 2:00 p.m. CST.
CME After Hours

- Extended-hours trading on GLOBEX runs from 2:30 p.m. to 4:00 p.m. dinner break and then back at it from 6:00 p.m. to 6:00 a.m. CST.
- Singapore International Monetary Exchange (SIMEX) offer interchangeable contracts.
- There’s other markets, but none are close to CME and SIMEX trading volume.
Basic Currency Futures Relationships

- *Open Interest* refers to the number of contracts outstanding for a particular delivery month.
- Open interest is a good proxy for demand for a contract.
- Some refer to open interest as the *depth* of the market. The *breadth* of the market would be how many different contracts (expiry month, currency) are outstanding.
## Reading a Futures Quote

<table>
<thead>
<tr>
<th>Expiry month</th>
<th>Open</th>
<th>Hi</th>
<th>Lo</th>
<th>Settle</th>
<th>Change</th>
<th>Lifetime High</th>
<th>Lifetime Low</th>
<th>Open Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept</td>
<td>.9282</td>
<td>.9325</td>
<td>.9276</td>
<td>.9309</td>
<td>+.0027</td>
<td>1.2085</td>
<td>.8636</td>
<td>74,639</td>
</tr>
</tbody>
</table>

- **Highest and lowest prices over the lifetime of the contract.**
- **Highest price that day**
- **Opening price**
- **Expiry month**
- **Closing price**
- **Daily Change**
- **Lowest price that day**
- **Number of open contracts**
Eurodollar Interest Rate Futures Contracts

- Widely used futures contract for hedging short-term U.S. dollar interest rate risk.
- The underlying asset is a hypothetical $1,000,000 90-day Eurodollar deposit—the contract is *cash settled*.
- Traded on the CME and the Singapore International Monetary Exchange.
- The contract trades in the March, June, September and December cycle.
Reading Eurodollar Futures Quotes

<table>
<thead>
<tr>
<th>EURODOLLAR (CME)—$1 million; pts of 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>July</td>
</tr>
</tbody>
</table>

Eurodollar futures prices are stated as an index number of three-month LIBOR calculated as $F = 100 - \text{LIBOR}$.

The closing price for the July contract is 94.68 thus the implied yield is 5.32 percent $= 100 - 98.68$

The change was .01 percent of $1$ million representing $100$ on an annual basis. Since it is a 3-month contract one basis point corresponds to a $25$ price change.
Options Contracts: Preliminaries

- An option gives the holder the right, *but not the obligation*, to buy or sell a given quantity of an asset in the future, at prices agreed upon today.

- Calls vs. Puts
  - Call options gives the holder the right, but not the obligation, to buy a given quantity of some asset at some time in the future, at prices agreed upon today.
  - Put options gives the holder the right, but not the obligation, to sell a given quantity of some asset at some time in the future, at prices agreed upon today.
Options Contracts: Preliminaries

- European vs. American options
  - European options can only be exercised on the expiration date.
  - American options can be exercised at any time up to and including the expiration date.
  - Since this option to exercise early generally has value, American options are usually worth more than European options, other things equal.
Options Contracts: Preliminaries

- **In-the-money**
  - The exercise price is less than the spot price of the underlying asset.

- **At-the-money**
  - The exercise price is equal to the spot price of the underlying asset.

- **Out-of-the-money**
  - The exercise price is more than the spot price of the underlying asset.
Options Contracts: Preliminaries

- **Intrinsic Value**
  - The difference between the exercise price of the option and the spot price of the underlying asset.

- **Speculative Value**
  - The difference between the option premium and the intrinsic value of the option.

\[
\text{Option Premium} = \text{Intrinsic Value} + \text{Speculative Value}
\]
Currency Options Markets

- PHLX
- HKFE
- 20-hour trading day.
- OTC volume is much bigger than exchange volume.
- Trading is in seven major currencies plus the euro against the U.S. dollar.
# PHLX Currency Option Specifications

<table>
<thead>
<tr>
<th>Currency</th>
<th>Contract Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australian dollar</td>
<td>AD50,000</td>
</tr>
<tr>
<td>British pound</td>
<td>£31,250</td>
</tr>
<tr>
<td>Canadian dollar</td>
<td>CD50,000</td>
</tr>
<tr>
<td>Deutsche mark</td>
<td>DM62,500</td>
</tr>
<tr>
<td>French franc</td>
<td>FF250,000</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>¥6,250,000</td>
</tr>
<tr>
<td>Swiss franc</td>
<td>SF62,500</td>
</tr>
<tr>
<td>Euro</td>
<td>€62,500</td>
</tr>
</tbody>
</table>
Currency Futures Options

- Are an option on a currency futures contract.
- Exercise of a currency futures option results in a long futures position for the holder of a call or the writer of a put.
- Exercise of a currency futures option results in a short futures position for the seller of a call or the buyer of a put.
- If the futures position is not offset prior to its expiration, foreign currency will change hands.
Basic Option Pricing
Relationships at Expiry

- At expiry, an American call option is worth the same as a European option with the same characteristics.
- If the call is in-the-money, it is worth $S_T - E$.
- If the call is out-of-the-money, it is worthless.

$$C_{aT} = C_{eT} = \text{Max}[S_T - E, 0]$$
Basic Option Pricing
Relationships at Expiry

- At expiry, an American put option is worth the same as a European option with the same characteristics.
- If the put is in-the-money, it is worth $E - S_T$.
- If the put is out-of-the-money, it is worthless.

$$P_{aT} = P_{eT} = \text{Max}[E - S_T, 0]$$
Basic Option Profit Profiles

\[ C_{aT} = C_{eT} = \text{Max}[S_T - E, 0] \]
Basic Option Profit Profiles

\[ C_{aT} = C_{eT} = \text{Max}[S_T - E, 0] \]
Basic Option Profit Profiles

\[ P_{aT} = P_{eT} = \text{Max}[E - S_T, 0] \]
Basic Option Profit Profiles

\[ C_{aT} = C_{eT} = \text{Max}[S_T - E, 0] \]
American Option Pricing Relationships

With an American option, you can do everything that you can do with a European option—this option to exercise early has value.

\[ C_{aT} \geq C_{eT} = Max[S_T - E, 0] \]

\[ P_{aT} \geq P_{eT} = Max[E - S_T, 0] \]
Market Value, Time Value and Intrinsic Value for an American Call

\[ C_{aT} \geq \text{Max}[S_T - E, 0] \]

Profit

\[ S_T - E \]

Market Value

Time value

Intrinsic value

Out-of-the-money In-the-money

\[ S_T \]

loss
European Option Pricing Relationships

Consider two investments

1. Buy a call option on the British pound futures contract. The cash flow today is \(-C_e\)

2. Replicate the upside payoff of the call by
   1. Borrowing the present value of the exercise price of the call in the U.S. at \(i_s\) The cash flow today is \(E/(1+i_s)\)
   2. Lending the present value of \(S_T\) at \(i_L\) The cash flow is \(-S_T/(1+i_L)\)
European Option Pricing Relationships

When the option is in-the-money both strategies have the same payoff.

When the option is out-of-the-money it has a higher payoff the borrowing and lending strategy.

Thus:

$$C_e \geq \max\left[ \frac{S_T}{(1 + i_x)} - \frac{E}{(1 + i_s)}, 0 \right]$$
European Option Pricing Relationships

Using a similar portfolio to replicate the upside potential of a put, we can show that:

\[ P_e \geq \max \left[ \frac{E}{(1+i_s)} - \frac{S_T}{(1+i_f)}, 0 \right] \]
Binomial Option Pricing Model

- Imagine a simple world where the dollar-euro exchange rate is $S_0(\$/\€) = $1 today and in the next year, $S_1(\$/\€)$ is either $1.1$ or $.90.$

<table>
<thead>
<tr>
<th>$S_0($/\€)$</th>
<th>$S_1($/\€)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1.10$</td>
</tr>
<tr>
<td>$1$</td>
<td>$.90$</td>
</tr>
</tbody>
</table>
A call option on the euro with exercise price $S_0(\$/\€) = $1 will have the following payoffs.

<table>
<thead>
<tr>
<th>$S_0($/\€)$</th>
<th>$S_1($/\€)$</th>
<th>$C_1($/\€)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1.10$</td>
<td>$.10</td>
</tr>
<tr>
<td>$1$</td>
<td>$.90</td>
<td>$0$</td>
</tr>
</tbody>
</table>
Binomial Option Pricing Model

- We can replicate the payoffs of the call option. With a levered position in the euro.

<table>
<thead>
<tr>
<th>$S_0$/€</th>
<th>$S_1$/€</th>
<th>$C_1$/€</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1.10$</td>
<td>$.10$</td>
</tr>
<tr>
<td>$.90</td>
<td></td>
<td>$0$</td>
</tr>
</tbody>
</table>
Binomial Option Pricing Model

Borrow the present value of $.90 today and buy € 1. Your net payoff in one period is either $.2 or $0.

<table>
<thead>
<tr>
<th>$S_0($/€)$</th>
<th>$S_1($/€)$</th>
<th>debt</th>
<th>portfolio</th>
<th>$C_1($/€)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1.10$</td>
<td>$-.90$</td>
<td>$.20$</td>
<td>$.10$</td>
</tr>
<tr>
<td>$1$</td>
<td>$.90$</td>
<td>$-.90$</td>
<td>$.00$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
The portfolio has twice the option’s payoff so the portfolio is worth twice the call option value.

<table>
<thead>
<tr>
<th>$S_0($/\€)$</th>
<th>$S_1($/\€)$</th>
<th>debt</th>
<th>portfolio</th>
<th>$C_1($/\€)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1.10$</td>
<td>$-.90$</td>
<td>$.20</td>
<td>$.10</td>
</tr>
<tr>
<td>$1$</td>
<td>$$.90$</td>
<td>$-.90$</td>
<td>$.00</td>
<td>$0$</td>
</tr>
</tbody>
</table>
Binomial Option Pricing Model

The portfolio value today is today’s value of one euro less the present value of a $.90 debt:

\[ S_0(\text{$/€}) \quad S_1(\text{$/€}) \quad \text{debt} \quad \text{portfolio} \quad C_1(\text{$/€}) \]

<table>
<thead>
<tr>
<th>S_0(\text{$/€})</th>
<th>S_1(\text{$/€})</th>
<th>debt</th>
<th>portfolio</th>
<th>C_1(\text{$/€})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$1.10</td>
<td>-$.90</td>
<td>$.20</td>
<td>$.10</td>
</tr>
<tr>
<td>$.90</td>
<td>-$.90</td>
<td>$.00</td>
<td>$0</td>
<td></td>
</tr>
</tbody>
</table>

\[ \frac{S_0 - \frac{.90}{1 + i_€}}{1.10 - \frac{.90}{1 + i_€}} \]
We can value the option as half of the value of the portfolio:

\[
C_0 = \frac{1}{2} \left( \frac{S_1 - d_0}{1 + r_d} \right)
\]

<table>
<thead>
<tr>
<th>(S_0($/€))</th>
<th>(S_1($/€))</th>
<th>debt</th>
<th>portfolio</th>
<th>(C_1($/€))</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$1.10</td>
<td>$0.20</td>
<td>$0.20</td>
<td>$0.10</td>
</tr>
<tr>
<td>$0.90</td>
<td>$0.90</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
Binomial Option Pricing Model

- The most important lesson from the binomial option pricing model is: the replicating portfolio intuition.

- Many derivative securities can be valued by valuing portfolios of primitive securities when those portfolios have the same payoffs as the derivative securities.
European Option Pricing Formula

- We can use the replicating portfolio intuition developed in the binomial option pricing formula to generate a faster-to-use model that addresses a much more realistic world.
European Option Pricing Formula

The model is

\[ C_0 = [F \times N(d_1) - E \times N(d_2)]e^{-r_S T} \]

Where

\( C_0 \) = the value of a European option at time \( t = 0 \)

\( F = S_t e^{(r_S - r_£)T} \)

\( r_S \) = the interest rate available in the U.S.

\( r_£ \) = the interest rate available in the foreign country—in this case the U.K.

\[ d_1 = \frac{\ln(F / E) + .5\sigma^2 T}{\sigma\sqrt{T}} \]

\( d_2 = d_1 - \sigma\sqrt{T} \)
European Option Pricing Formula

Find the value of a six-month call option on the British pound with an exercise price of $1.50 = £1
The current value of a pound is $1.60
The interest rate available in the U.S. is $r = 5\%$.
The interest rate in the U.K. is £$r = 7\%$.
The option maturity is 6 months (half of a year).
The volatility of the $$/£$ exchange rate is 30\% p.a.
Before we start, note that the intrinsic value of the option is $.10—our answer must be at least that.
European Option Pricing Formula

Let’s try our hand at using the model. If you have a calculator handy, follow along.

First calculate

\[ F = S_t e^{(r_s - r_f)T} = 1.50e^{(.05-.07)0.50} = 1.485075 \]

Then, calculate \( d_1 \) and \( d_2 \)

\[
d_1 = \frac{\ln(F / E) + .5\sigma^2 T}{\sigma \sqrt{T}} = \frac{\ln(1.485075/1.50) + .5(0.4)^2 \cdot 5}{.4\sqrt{.5}} = 0.106066
\]

\[
d_2 = d_1 - \sigma \sqrt{T} = 0.106066 - .4\sqrt{.5} = -0.176878
\]
European Option Pricing Formula

\[ F = 1.485075 \]
\[ d_1 = 0.106066 \]
\[ d_2 = -0.176878 \]

\[ N(d_1) = N(0.106066) = 0.5422 \]
\[ N(d_2) = N(-0.1768) = 0.4298 \]

\[ C_0 = [F \times N(d_1) - E \times N(d_2)]e^{-r_sT} \]

\[ C_0 = [1.485075 \times 0.5422 - 1.50 \times 0.4298]e^{-0.05 \times 0.5} = 0.157 \]
Option Value Determinants

<table>
<thead>
<tr>
<th></th>
<th>Call</th>
<th>Put</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Exchange rate</td>
<td>+</td>
</tr>
<tr>
<td>2.</td>
<td>Exercise price</td>
<td>–</td>
</tr>
<tr>
<td>3.</td>
<td>Interest rate in U.S.</td>
<td>+</td>
</tr>
<tr>
<td>4.</td>
<td>Interest rate in other country</td>
<td>+</td>
</tr>
<tr>
<td>5.</td>
<td>Variability in exchange rate</td>
<td>+</td>
</tr>
<tr>
<td>6.</td>
<td>Expiration date</td>
<td>+</td>
</tr>
</tbody>
</table>

The value of a call option $C_0$ must fall within

$$\max (S_0 - E, 0) \leq C_0 \leq S_0.$$  

The precise position will depend on the above factors.
Empirical Tests

The European option pricing model works fairly well in pricing American currency options.

It works best for out-of-the-money and at-the-money options.

When options are in-the-money, the European option pricing model tends to underprice American options.
End Chapter Nine